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Car accidents and number of stopped cars due to road blockage on a one-lane highway

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Abstract. Within the framework of a simple model of car traffic on a one-lane highway, we study the probability of the occurrence of car accidents when drivers do not respect the safety distance between cars, and, as a result of the blockage during the time T necessary to clear the road, we determine the number of stopped cars as a function of car density. We give a simple theory in good agreement with our numerical simulations.

1. Car accidents

In the past few years, many highway traffic models formulated in terms of cellular automata have been studied, both in one [1–5] and two dimensions [6, 7]. For the one-dimensional case, several topological variations of the basic model have been proposed, including road crossing [10], road with junction [11] and two-lane highway [12–14]. Recently, experimental features and complex spatiotemporal structures of real traffic flows have been investigated [15, 16].

One of the simplest models is defined on a one-dimensional lattice of L sites with periodic boundary conditions. Each site is either occupied by a vehicle, or empty. The velocity of each vehicle is an integer between 0 and $v_{\max}$. If $x(i,t)$ denotes the position of the ith car at time t, the position of the next car ahead at time t is $x(i+1,t)$. With this notation, the system evolves according to a synchronous rule given by

$$x(i,t+1) = x(i,t) + v(i,t+1) \tag{1}$$

where

$$v(i,t+1) = \min(x(i+1,t) - x(i,t) - 1, x(i,t) - x(i,t-1) + a, v_{\max}) \tag{2}$$

is the velocity of car i at time $t+1$. $x(i+1,t) - x(i,t) - 1$ is the gap (number of empty sites) between cars i and $i+1$ at time t, $x(i,t) - x(i,t-1)$ is the velocity $v(i,t)$ of car i at time t, and a is the acceleration. $a = 1$ corresponds to the deterministic model of Nagel and Schreckenberg [1], while the case $a = v_{\max}$ has been considered by Fukui and Ishibashi [17]. In this last case, the evolution rule can be written as

$$x(i,t+1) = x(i,t) + \min(x(i+1,t) - x(i,t) - 1, v_{\max}). \tag{3}$$

This is a cellular automaton rule whose radius is equal to $v_{\max}$. The case $a < v_{\max}$ is a second-order rule, that is, the state at time $t+1$ depends upon the states at times t and $t-1$.

We studied the probability for a car accident to occur when drivers do not respect the safety distance. More precisely, if at time t, the velocity $v(i+1,t)$ of car $i+1$ was positive, expecting this velocity to remain positive at time $t+1$, the driver of car i increases the safety velocity $v(i, t+1)$ given by (2) by one unit, with a probability p. The evolution rule (1) is then replaced by

$$\text{if } v(i+1, t) > 0 \qquad \text{then } x(i, t+1) = x(i, t) + v(i, t+1) + \Delta V \tag{4}$$

where ΔV is a Bernoulli random variable which takes the value 1 with probability p and zero with probability $1-p$. If $v(i+1, t+1) = 0$, it is clear that this careless driving will result in an accident.

When the car density, ρ, is less than the critical car density $\rho_c = (1+v_{\max})^{-1}$, the average number of empty sites between two consecutive cars is larger than $v_{\max}$, the fraction, n_0, of stopped cars is zero and no accident can occur. If $\rho > \rho_c$, the average velocity is less than $v_{\max}$, n_0 increases with ρ and careless driving will result in a number of accidents. This number will, however, go to zero for $\rho = 1$, since, in this case, all cars are stopped. The probability for a car accident to occur should, therefore, reach a maximum for a car density, ρ, between ρ_c and 1.

Neglecting time correlations, we may determine an approximate probability for an accident to occur. Let n be the number of empty sites between cars i and $i+1$ at time t. If the three conditions

$$0 \leqslant n \leqslant v_{\max} \qquad v(i+1, t) > 0 \qquad v(i+1, t+1) = 0 \tag{5}$$

are satisfied then car i will cause an accident at time $t+1$, with a probability p. Therefore, the value, P_{as}, of the probability *per site* and *per time step* for an accident to occur is given by

$$P_{as} = pn_0(1-n_0)\sum_{n=0}^{v_{\max}} \rho^2(1-\rho)^n = p\rho(1-(1-\rho)^{v_{\max+1}})n_0(1-n_0). \tag{6}$$

Dividing by ρ, one obtains the probability *per car* and *per time step* for an accident to occur

$$P_{ac} = p\rho(1-(1-\rho)^{v_{\max+1}})n_0(1-n_0). \tag{7}$$

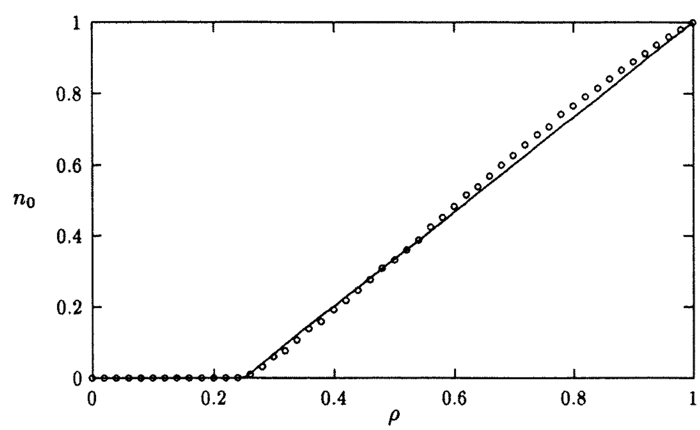

Figure 1. The fraction of stopped cars, n_0, as a function of the car density, ρ, for $v_{\max} = 3$ and $a = 1$. The full line represents the linear approximation $n_0 = (\rho - \rho_c)/(1 - \rho_c)$.

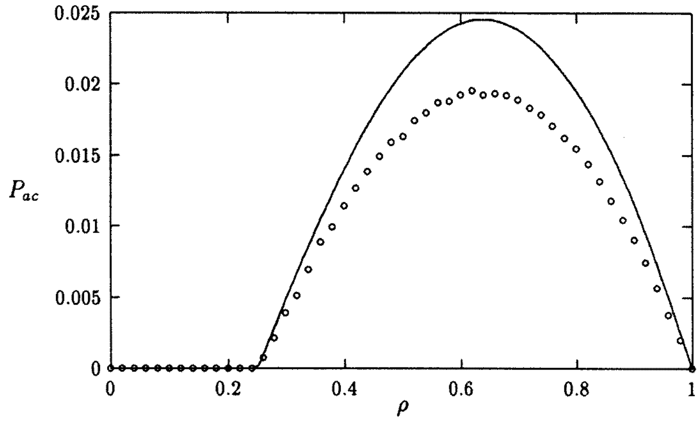

Figure 2. The probability, P_{ac}, per car and per time step for an accident to occur as a function of the car density, ρ, for $v_{\max} = 3$ and $a = 1$. The full curve corresponds to the approximation given by (9).

The simplest approximate expression for the fraction n_0 of stopped cars as a function of car density, satisfying the above-mentioned conditions, is

$$n_0 = \frac{\rho - \rho_c}{1 - \rho_c}. \tag{8}$$

This approximation is rather crude. In particular, it neglects the fact that n_0 should depend upon $v_{\max}$. However, as shown in figure 1, for $v_{\max} = 3$ and $a = 1$, this linear approximation is in rather good agreement with our numerical results. Substituting (8) into (7) yields

$$P_{ac} = p\rho(1 - (1 - \rho)^{v_{\max+1}}) \frac{(\rho - \rho_c)(1 - \rho)}{(1 - \rho_c)^2}. \tag{9}$$

Figure 2 represents the probability, P_{ac}, as a function of ρ determined numerically and its value given by (9).

2. Stopped cars due to a blockage

As a result of an accident (or any other cause such as road works), the traffic is blocked during the time, T, necessary to clear the road. For a given T, the number, $N(\rho)$, of blocked cars is clearly an increasing function of the car density ρ. To determine the expression of N, we shall distinguish between two regimes.

If $\rho \leqslant \rho_c$, the average car velocity is $v_{\max}$. Since the average number of empty sites between two consecutive cars is equal to

$$d(\rho) = \frac{(1 - \rho)}{\rho} \tag{10}$$

the line of stopped cars increases by one unit during the time interval

$$\frac{v_{\max}}{d(\rho)}. \tag{11}$$

Hence, during the time T the number N of blocked cars is given by

$$N(\rho) = T\frac{\rho(1 - \rho_c)}{(1 - \rho)\rho_c}. \tag{12}$$

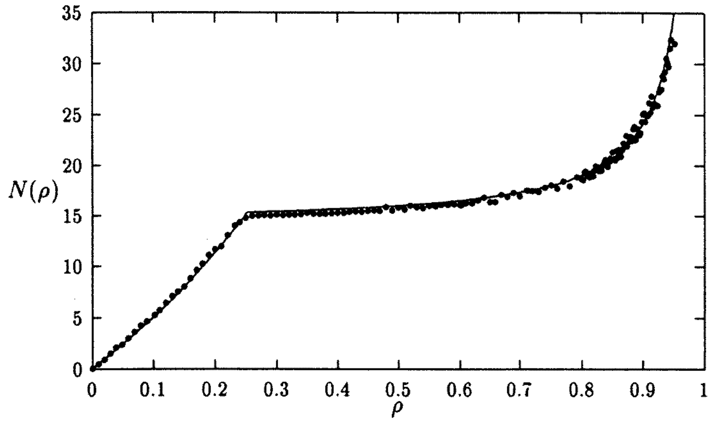

Figure 3. The average number of blocked cars as a function of the car density ρ. The full curve corresponds to the approximate expressions (12) and (13).

For $\rho = \frac{1}{2}$ the average number of empty sites between two consecutive cars is equal to $d(\frac{1}{2}) = 1$, and the average number of blocked cars increases by one unit at each time step. $N(\rho)$ increases, therefore, from T to $T + 1$ when ρ increases from ρ_c to $\frac{1}{2}$ (note that the first blocked site is occupied). When $\rho > \frac{1}{2}$, $d(\rho) < 1$, the average number of blocked cars is then given by

$$N(\rho) = T + \frac{1}{d(\rho)} = T + \frac{\rho}{1 - \rho}. \tag{13}$$

Figure 3 represents the average number of blocked cars as a function of the car density ρ. The agreement with the approximate expressions of $N(\rho)$, given by (12) and (13), is very good.

References

[1] Nagel K and Schreckenberg M 1992 *J. Physique* I **2** 2221
[2] Nagel K and Herrmann H J 1993 *Physica* **199A** 254
[3] Schadschneider A and Schreckenberg M 1993 *J. Phys. A: Math. Gen.* **26** L679
[4] Schreckenberg M, Schadschneider A, Nagel K and Ito N 1995 *Phys. Rev.* E **51** 2939
[5] Vilar L C Q and de Souza A M C 1994 *Physica* **211A** 84
[6] Biham O, Middleton A A and Levine D 1992 *Phys. Rev.* A **46** R6124
[7] Tadaki S and Kikuchi M 1994 *Phys. Rev.* E **50** 4564
[8] Tadaki S and Kikuchi M 1995 *J. Phys. Soc. Japan* **64** 4504
[9] Molera J M, Martinez F C, Cuesta J A and Brito R 1995 *Phys. Rev.* E **51** 175
[10] Ishibashi Y and Fukui M 1996 *J. Phys. Soc. Japan* **65** 2793
[11] Benjamin S C, Johnson N F and Hui P M 1996 *J. Phys. A: Math. Gen.* **29** 3119
[12] Nagatani T 1994 *J. Phys. Soc. Japan* **63** 52
[13] Nagatani T 1996 *J. Phys. A: Math. Gen.* **29** 6531
[14] Rickert M, Nagel K, Schreckenberg M and Latour A 1996 *Physica* **231A** 534
[15] Kerner B S and Rehborn H 1996 *Phys. Rev.* E **53** R1297
[16] B K S and Rehborn H 1996 *Phys. Rev.* E **53** R4275
[17] Fukui M and Ishibashi Y 1996 *J. Phys. Soc. Japan* **65** 1868